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SYNTHESIS OF ENVIRONMENTAL SOUND TEXTURES BY ITERATED NONLINEAR FUNCTIONS

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ABSTRACT

In previous work, a class of digital sound synthesis methods was introduced using iterated nonlinear functions [1][2][3][4]. Within the phase space of any method in the class, we encounter regions of special interest where signals have peculiar self-similar structures (waveforms of multiple fractal contours) [5][6]. Due to the system dynamics, emergent properties in the output sound signal result into acoustic turbulences and other textural sound phenomena. Parallel work was pursued, both in computer music research [7] and in the auditory display of experimental data, using chaotic oscillators [8] (nonlinear pendulums are also illustrated in [9]).

This paper discusses the use of iterated nonlinear functions in the modelling of the perceptual attributes in complex auditory images. Based on the chaotic dynamics in such algorithms, it is possible to create textural and environmental sound effects of a peculiar kind, hardly obtained with other methods. Examples include sound textures reminiscent of rains, thunderstorms and more articulated phenomena of acoustic turbulence.

This research opens to new experiments in electroacoustic music and the creation of synthetic, but credible, auditory scenes in multimedia applications and virtual reality.

1. SURVEY OF FUNCTIONAL ITERATION SYNTHESIS

Consider these definitions:

$A \subset \Re$ a set of "init values" for some iterated map process;
$G \subset \Re^m$ a set of parameters for the particular map considered;
$B \subset \Re$ a set of samples in the digital signal finally generated;

and consider the cartesian product $A \times G \subset \Re \times \Re^m$. Let F be a map defined as

$$F: A \times G \to B, \quad (x, \{a_i\}) \to F(x; \{a_i\})$$

where $(\{a_i\} \equiv a_1, a_2, ..., a_m)$. This represents a parameter-dependent function which maps from A to B, with a_i as a set of time-changing parameters. By fixing a set of *m* real parameters (a point in G) we have:

$$f: A \to B, \quad x \to f(x)$$

where $f(x) \equiv F(x; a_1...a_m)$. If $B \subset A$, then we can implement an iterated map process of by iteratively applying *f* to itself for *n* times:

$$f^n(x) \equiv f(f(\cdots f(x) \cdots)) \equiv (f \circ f \circ \cdots \circ f)(x)$$

(the symbol ° denotes "composition", so here we should speak of "*n* compositions of *f* with *x*"). Finally, considering g_i a sequence of points in G, and provided that $x_{0,i} \in A$ and $g_i \in G$, we get a sequence of maps f_i. A discrete time series is generated where each sample is the *n*-th iterate of the same function using different parameters (parameters can be updated by some other control function):

$$x_{n,i} = f^n_i(x_{0,i}) = f_i(f_i(...(f_i(x_{0,i}))...)) \quad (1)$$

This is a general template for any method of *Functional Iteration Synthesis* (FIS) [2]. For interesting sounds, any well-chosen nonlinear *f* can be adopted, and smoothly changing control functions should be used to update the parameters value at each iterated process (= at each sample). Indeed, FIS is less a specific sound synthesis technique than a class of methods sharing the iteration of nonlinear maps as the basic operation. Obviously, every particular FIS model has its own characteristics depending on the particular function adopted. However, I should stress that the crucial point here is more with the process of iteration than the function itself. As Mitchell Feigenbaum observed, «[…] *precisely because the same operation is reapplied* [...] *self-consistent patterns might emerge where the consistency is determined by the key notion of iteration and* not *by the particular function performing the iterates*» [10].

2. THE "SINE MAP" MODEL

Let's now consider the following map:

$$F: [-\pi/2, \pi/2] \times [0, 4] \to (-1, 1), \quad (x, r) \to \sin(rx)$$

The iterated discrete form is

$$x_{n,i} = \sin(r_i\, x_{n-1,i}) \quad (2)$$

and represents a special case of (1). It is often found in the literature on " chaos theory", and represents a very simple algorithm to implement with a computer. There, A = [$-\pi/2$, $\pi/2$] because larger intervals for the init values would result into iterate trajectories already included when the process starts from within [$-\pi/2$, $\pi/2$] (clearly, this is due to the periodicity of the sine function). The first iterate always falls in the interval [-1, 1], completely covered by $sin(rx)$ for x_0 in [$-\pi/2$, $\pi/2$] and $r \geq 1$. In (2), also, G = [0, 4] because larger values for r would provide results similar to, if not identical with, those provided by $r < 4$.

The dynamics of the iterated sine map could be graphically rendered by tracing the n-th iterate of some x_0 while linearly increasing r (this is the usual method for tracing the well-known "bifurcation diagram", a visual characterizion of nonlinear dynamics). Figure 1 shows the 5^{th} iterate of the sine map model, in the region r=[3,4], x_0= [0.1, 0.1] (a single "slice" in the phase space). Figure 2 shows the 7^{th} iterate in the very same region.

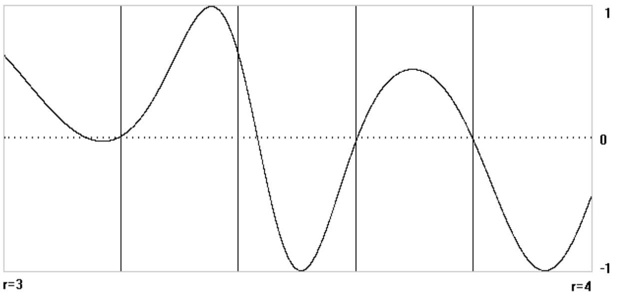

figure 1

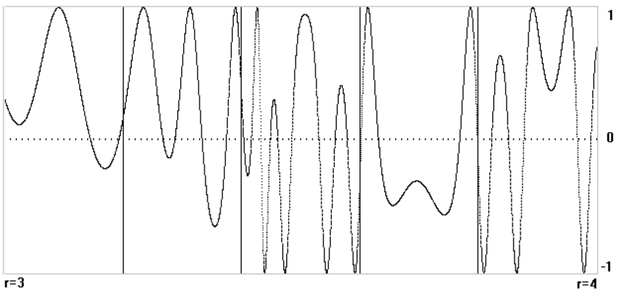

figure 2

As is expected, higher iterates yield richer (more active and dynamical) trajectories. These oscillating trajectories, and the numerical relationship among them and their derivatives, have not been specifically addressed by any scientific study.

As a matter of fact, we find ourselves in a territory where little help is provided by purely analytical means, and research work turns to more explorative, experimental methodologies. When a higher iterate is calculated, the init value x_0 is soon *forgotten*: one cannot tell, by any analytical means, where in the interval [$-\pi/2$, $\pi/2$] the iteration process was started. Transients disappear and the final plot for the process becomes coincident with the now popular bifurcation diagram, as shown in figure 3 (where 10 iterates are plotted, ranging from the 90^{th} to the 100^{th}).

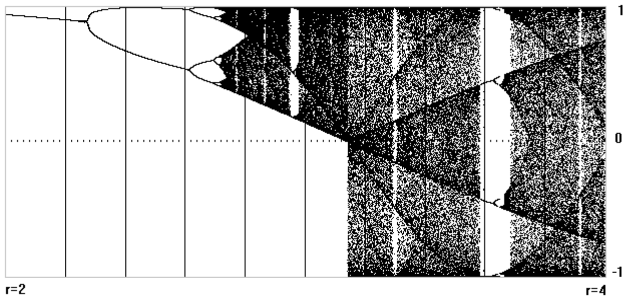

figure 3

3. EXPLORING THE PHASE SPACE

The output of the particular FIS model depends on the orbit traced in the iterated process phase space [$-\pi/2$, $\pi/2$] $\times$ [0, 4] or a region within it. Figure 4 is an approximate graphical rendition of the region [0,1] $\times$ [2,4] as relative to the 5^{th} iterate. In general, with any FIS model, the output signal is coincident with the path traced by "stepping through" such an uneven terrain, with its hills and valleys.

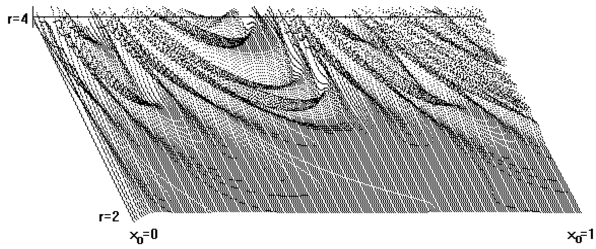

figure 4

Indeed, for any monoparametric map, the orbit is defined as the coupling of the two series, r_i and $x_{0,i}$. The time series $x_{n,i}$ is then a digital audio signal resulting from that coupling. The waveform shape is determined, among other things, by the orbital velocity across the phase space.

Broadly speaking, r determines the kind of waveform shape in the sound signal, ranging from very smooth curves (e.g. r = 2) to more intricate oscillations (e.g. r = 4). Many possibilities are included in between, as due to either bi- or multi-periodic cycles (limit-cycle attractors), or wild aperiodic oscillations (strange attractors). The latter signals always include some continuing frequency- or phase-modulation component.

On the other hand, x_0 determines the actual sample values. Slight changes in x_0 result in signals maybe overlapping at the outset but then gradually shifting apart from one another (a typical case of "dependency on the initial conditions").

The number of iterations, n, significantly affects the spectrum bandwidth. With larger values the signal goes through wider and wider oscillations, which might even have a fractal shape (signal contours are repeated at various time-scales in the signal). Eventually the spectrum gets denser and noise is obtained, perhaps with a broad but limited band. In most cases, n = 9 already returns some articulated noise texture. The bandwidth also depends on the rate by which we are sampling the orbit in the phase space (i.e., on the actual step in walking across the system phase space).

During the synthesis we may (a) change r and keep x_0 constant; (b) change x_0 and keep r constant; and (c) change both r and x_0 (the iterate order, n, cannot change, as that would determine audible discontinuities). These changes create, respectively, (a) varying contours in the signal waveform (dynamical spectra); (b) different signal waveforms *of the same kind* (nearly-constant spectrum); and (c) a mixture often heard as articulated sound textures in a constant flux of change, including sudden “pauses” (sub-audio frequencies and DC).

If either r or x_0 are driven with a periodic driving function, we get a closed orbit in the phase space. Therefore, the sound include some periodicity, with audible patterns either in the sub-audio (rhythms) or audio range (pitches) (depending on orbital velocity). Here, I do not consider this possibility, as I would like to focus on the peculiar opportunity of exploiting the narrow-band, articulated noises created by the iterated sine map model in its “natural” condition (i.e. not forced to periodic beaviour).

Here, I cannot discuss the computer implementation of FIS. A very simple C language implementation of the iterated sine map model is in [2] (audio examples can be downloaded from *http://www.swets.nl/jnmr/vol25_1.html#discipio25.1*). A Csound implementation is discussed in [11], together with commented code examples. The real-time implementation with Kyma (® SymbolicSound) is discussed in documents publically available at *ftp://shout.net/pub/symsound/kymasnds/Agostino_DiScipio/*. In the latter case, the iterated sine map is viewed as a generalization of the waveshaping synthesis method introduced in [12].

4. SYSTEM NONLINEARITY AND PERCEPTUAL MODELLING OF TEXTURAL AUDITORY IMAGES

The time-dependent relationship between r_i and $x_{0,i}$ is crucial, and radically changes when different iterates (n) are considered. In a typical situation, one cannot modify any of the three values (r, x_0 and n) without causing, as a side-effect, a change in the way the others affect the overall result.

This phenomenon reflects reveals the non-integrability of chaotic systems to the ear. We are only left with the possibility of a *qualitative* characterization of the interdependency among parameter values. Clearly it is impossible to predict the precise output of any configuration of values. Put the other way round, it is impossible to fix the values to get a specific target sound, especially when the model is not forced to periodic behaviour. The exploration of the phase space, and the exploration of the parameter space as an ideal (but far from obvious) mapping of the perceptual space in the audible effects, should be left to interactive experiments (this is why a highly interactive real-time implementation is necessary in musical contexts [4][7]). Only *a posteriori* it becomes possible to fix some values and delimit phase space regions of special interest. That is the case with the sound examples illustrated in the following section.

What is significant in this approach, is that it provides the opportunity for a perceptual modelling of textural sound events based on a time-domain synthesis method. It is common understanding that sonic phenomena of textural nature – such as the sound(s) of the rain, cracking of rocks and icebanks, thunders, electrical intermittent noises, the sound(s) of the wind, various kinds of “sonorous powders”, burning materials, rocky sea shores, certain kinds of insects, etc. – are best modelled by microstructural time-based design strategies, rather than spectral modelling strategies (see examples in [13][14][15], and musical considerations in [16][17], where granular synthesis methods were employed). Finally, it seems very appropriate that the effects generated with FIS require wholistic and largely indeterministic controls.

5. SOUND TURBOLENCES AND THUNDERSTORMS

It should be stressed that none of the output sound effects generated with the iteration of nonlinear functions is similar to some white noise submitted to filtering, however complex the filter system can be. The rate of change in the waveform of white-noise, or even that of $1/f^2$ noise is too fast for the ear and finally result into a sustained, non-articulated sound.

Sounds obtained with the iterated sine map model, instead, feature an internal articulation of their own, with random fluctuations which are anyway slower than white noise. Phase-modulations and amplitude curves are somehow “built in”, they are innate to the system dynamics, and result into a micro-level activity which is useful in order to approximate environmental sound textures. In a real auditory scene, moving sources and reflecting surfaces cause all sorts of interferences among signals, which are for the ear as a peculiar element of realistic sound ambience. The rate of change in the amplitude of the generated signal usually ranges between shorter than 1/10th of a second to seconds. Which means, also, that applications can rely on the emergence of amplitude shapes “internal” to the sound (only some fade-in and fade-out is necessary to avoid discontinuities when the sound starts and ends).

The output signals obtained by visiting at random the phase space of the model, are heard as acoustical turbulences, very low-frequency (and even sub-audio frequency) rumbling sonorities. The actual frequency contents depend on the orbital velocity. Higher velocities result into larger bandwidths. Some effective but indeterministic control on the bandwidth is possible with carefully studied control functions or signals for r and x_0.

Taken *per se*, the low-frequency narrow-band noise of the iterated sine map model is a good starting point to model the perception of textures of, e.g., boiling water, sulphureous or volcanic areas, and the wind flapping against thin but large plastic or aluminum plates.

With higher-order iterates (e.g. $n > 8$), the signal waveform shows interesting correlations at multiple time-scales. Energy is scattered around in small, phase-modulated wavepackets of different lengths and amplitude (the sound is rather reminiscent of a large fire). With a high-order band-pass filter, it would be possible to isolate the energy at a particular time-scale. The effect would be that of a granular texture with a specific density of sonic microevents. However, more interesting is to submit the output of the iterated process to a simple 2nd order high-pass filter: by gradually lowering the cut-off frequency, sound droplets of larger and larger size are introduced, similar to the accumulation of raindrops when a rain shower is starting.

With rain-like sounds, it is important to introduce some perceptual clue of the different surfaces impacted upon by the raindrops. This can be done by choosing the appropriate iterate, n. Upon listening, larger values of n result into more resonant materials, while smaller values are more reminiscent of "deaf", very compact surfaces. Furthermore, by visiting the extreme regions in the system phase space ($r \leq 4$), we can control the perceived size of droplets, resulting either into a drizzling or a driving rain. As the droplet size gets larger, eventually a hailstorm is approached.

In this process, taking the cut-off frequency close to zero is like letting all energy at the different time-scales in the signal pass the filtering, resuming the characteristic turbulence of the "natural" state of the iterated sine map model. However, this time the sound texture is phase-modified by the high-pass, and might result into a surprisingly realistic thunderblot.

With just a few changes in the parameters value, that which so far was described as (various types of) rain or hail, becomes more similar to the sound of frying oil, or to boiling water burbles. In order to come closer to such sound effects, the underlying model may be re-configured either using an iterated function other than a sine, or adding some random numbers in the iterated sine map at each sample computation.

In musical contexts, I tend to use a large palette of finely-tuned parameter configurations in the synthesis, and to compose the emerging textures and gestures with the help of some higher-level iterated function system. That can be described an instance of algorithmic composition, but it uses chaotic textural materials instead of musical notes as the basic musical units. In a recent work of mine, *Natura allo specchio*,[1] the strategy was to create utterly synthetic sound environments that could eventually be perceived as naturalistic, keeping the listener in an ambivalent situation where the sound is overtly artificial and still preserves something (a dynamical behaviour) that is realistically perceived as proper to natural sonic ambiances (the compositional idea has to do with the interplay of Nature and Culture as commented in Shakespeare's *The Tempest*).

6. PRELIMINARY CONCLUSIONS

In this paper, I overviewed an approach on sound synthesis described as *Functional Iteration Synthesis*, and illustrated the particular case of the iterated sine map model. I meant to show that the nonlinear dynamics proper to such methods is appropriate for audio effects of textural nature. Ideally, this research takes on an older issue in computer music research, i.e. the simulation of natural sounds based on psychoacoustical considerations. However, while the work of, a.o., Jean-Claude Risset was primarily oriented towards the computer simulation of musical instruments, the present work is oriented towards the synthesis of credible auditory scenes and their global emergent properties. This is useful when creating special digital audio effects for musical applications and sophisticated audio components in multimedia and virtual reality applications.

The subject discussed here represents a new area of concern in the field of digital sound synthesis, and is linked to an "ecological" approach on auditory perception [13]. Due to the nonlinear dynamics of the synthesis models and the breath-taking complexity in the modelled phenomena, it is clear that more systematic research work is needed. Hopefully, the present paper delineated technological and scientifical aspects of relevance to the subject.

REFERENCES

[1] Di Scipio, A. "An Overview of Digital Sound Synthesis by Models of Nonlinear Dynamical Systems", *Sixth IMA Conference on Control: Modelling, Computation, Information*, Institute of Mathematics and its Applications, Sounthend-on-sea, 1992.

[2] Di Scipio, A. and Prignano, I. "Synthesis by Functional Iterations. A Revitalization of Nonstandard Synthesis", *Journal of New Music Research*, 25(1), 1996.

[3] Prignano, I. "Sintesi di eventi sonori complessi mediante iterazioni funzionali", Proc. *XI CIM*, AIMI, 1995.

[4] Di Scipio, A. "Composing with Iterated Nonlinear Functions in Interactive Environments", *Proc. XII CIM*, AIMI, 1998.

[5] May, R. "Simple Mathematical Models with Very Complicated Dynamics", *Nature*, 261, 1977.

[6] Collet, P. and Eckmann, J.P. *Iterated Maps on the Interval as Dynamical Systems*, Birkhauser, 1980.

[7] Choi, I. "A Chaotic Oscillator as a Musical Signal Generator in an Interactive Performance System", *Journal of New Music Research*, 26(1), 1997.

[8] Parker, T.S. and Chua, L.O. "Chaos: a Tutorial for Engineers", *Proc. IEEE*, 75(8), 1987.

[9] Campbell, D. "Nonlinear Science from Paradigms to Practicalities", *Los Alamos Science*, 15, 1987.

[10] Feigenbaum, M "Universal Behavior in Nonlinear Systems", *Los Alamos Science*, 1, 1980.

[11] Di Scipio, A. "Lecture on Sound Synthesis by Iterated Nonlinear Functions", in (R.Bianchini and A.Cipriani, eds.) *Virtual Sound*, Contempo, 1999.

[12] Arfib, D. "Digital Synthesis of Complex Spectra by means of Multiplication of Non-linear distorted sine waves", *Journal of the Audio Engineering Society* , 27(10), 1979.

[13] Bregman, A. *Auditory Scene Analysis*, MIT Press, 1990.

[14] Schumakler, M. and Gilden, D. "Auditory Perception of Fractal Contours", *Journal of Experimental Psychology*, 19(3), 1993.

[15] Warren, R. and Verbrugge, R. "Auditory Perception of Breaking and Bouncing Events: a Case Study of Ecological Acoustics", *Journal of Experimental Psychology. Human Perception and Performance*, 10(5), 1984.

[16] Di Scipio, A. "Micro-time Sonic Design and the Formation of Timbre", *Contemporary Music Review*, 10(2), 1994.

[17] MacKay, J. "On the Perception of Density and Stratification in Granular Sonic Textures", *Interface-Journal of New Music Research*, 13, 1984.

[1] This music is available on a compact disc issued by the *First Iteration Conference*, a conference on generative models in the arts, held in December 1999 at Monash University, in Melbourne (see conference site *http://www.cs.monash.edu.au/~iterate/*)